1 Lab Mechanics

The labs will be in Ho105. Dates and times for each group will be announced appropriately.

1.1 Report

In this laboratory exercise we will investigate one of the fundamental properties of quantum mechanics: superposition. The lab will have two parts. In the first part you will get acquainted with the apparatus. In the second part you will take data. You will be required to file a report that contains the following:

- Experimental layout.
- Answers to all the questions.
- Any graphs that are requested.
- Conclusions.

1.2 Notebook

Each person have a composition-type lab notebook (small size, grid paper preferably). Annotations should be written with a pen (not pencil). The lab should be NEAT and READABLE. Errors should be corrected by crossing the incorrect values clearly and writing new ones in a separate spot (not on top of the old ones!). For a given lab, the notebook should contain the following entries:

- Date.
- General information about the lab and its goal.
• A diagram of the setup, with as many details as possible. We will be using mostly the same equipment (different arrangements) for all the labs, so the information can be taken as cumulative. That is, once you write the specifications of some piece of equipment (e.g., wavelength of the laser), you do not need to enter it again at a later time unless it has changed.

• Information about each automated scan:
  
  – Scan number. (Create a scan-numbering system for the entire semester.)
    All scans should be numbered sequentially. Each group may have their own numbering system. For example, scan number 12 (overall) recorded in the third lab could be named “L3SC12.”
  
  – Information relevant to each scan (beginning, ending and increment). If the scan is a repeat of a previous one, you do not need to enter all the information.
  
  – Any other comment about the data, as it comes out, that seems relevant to the analysis that will be done later.
  
  – Any problems (e.g., “clumso” bumped the table in mid scan so the pattern has a “bad” point).

2 Conceptual Background

The apparatus has three components: the optical system, the electronic system, and the data acquisition system. All three components gear around the concept of doing experiments one photon at a time. This is accomplished by first creating pairs of photons. We then detect two separate channels of photons, but record only photon pairs (i.e., photons with partners). We insure the latter by detecting photons in “coincidence.”

2.1 Parametric Down-conversion

The optical system contains a “pump” laser, with a wavelength of 402.36 nm (it is a deep blue color). This beam is sent onto a non-linear crystal. If certain conditions are met then the process of spontaneous parametric down-conversion will occur, with a probability of about $10^{-10}$. This process consists of generating two photons from an incoming pump photon. In our experiment the crystal is set up so that the down-converted photons have the same energy and come out of the crystal at $\pm 3^\circ$.

**Question 1** What is the wavelength of the down-converted photons?
The down-converted photons are split into two paths, as shown in Fig. 1. In one path they go to a detector, and in the other path they go through some optical elements and then to a detector.

Another detail about the down-conversion is that the polarization of the down-converted photons is perpendicular to that of the pump photon.

Question 2 How could the light from the pump beam be stopped from reaching the detectors? (Check your answer by inspecting the actual setup.)

2.2 Detection

The optical component of apparatus ends in the detectors. These are “avalanche photodiodes.” They detect single photons with an efficiency of about 60% (that is very good!). However, if the number of photons incident on the detector exceeds $10^8$ per second they burn. The price of the detector module is $12,000, and the lag time in ordering one ranges between six and nine months. Therefore, if we burn it will have disastrous consequences.

Question 3 Suppose that the room is illuminated with the standard fluorescent lights. If the detector is exposed to the room light, it receives about $10^{-4}$ W. If we assume that the light is made of photons of wavelength 500 nm (green, for simplicity), how many photons are incident on the detector per unit time?

For this reason, we do the experiments in full darkness. Make sure that the power to the detectors is off while the lights of the room are on.
2.3 Electronics

In a future lab we will go into the details of how the detectors get wired. For this lab everything will be wired for you. Suffices to say that the detectors put out rectangular pulses every time that a photon is detected. The pulses from the detectors go to coincidence detection electronics, as shown in Fig. 2. The outputs of the electronics are the “singles” counts from each detector and the coincidences. All three signals are sent to counters and to the computer for data acquisition.

![Figure 2: Block diagram of the electronic system.](image)

2.4 Data acquisition

The data is acquired by the computer via a program written in Labview. It is straightforward to use. It has a front panel that shows four graphs. In this lab we will use only three graphs. Each graph displays the count sequences as a function of the position of the piezo. After you are done with each scan, you will be prompted for a file name. Create your own directory on the desktop and save all your files there throughout the semester. The files will have five columns of numbers representing the piezo voltage, idler singles counts, signal singles counts, coincidence counts and extra detector counts (which we do not use). The bottom two lines have the scan data. The data file can later be imported into Excel for graphing.

3 Photonic polarization

The instructor will give you an introduction to the apparatus. Once we are ready to detect single photons we turn the lights off and then we plug the detectors. Your tasks are the following:
3.1 Polarizers

A polarizer is easily understood classically. It provides the following function: when a linearly polarized electromagnetic wave is incident on a polarizer, the transmitted wave has an amplitude equal to the component of the field along the direction of the transmission axis of the polarizer and a polarization direction along that axis. If the Electric field vector of a wave of amplitude $E_0$ oscillates along a plane that forms an angle $\theta$ with the transmission axis of the polarizer, then the amplitude of the transmitted wave is

$$E_T = E_0 \cos \theta.$$ \hspace{1cm} (1)

Since the intensity of the wave is proportional to the square of the amplitude, then the intensity of the transmitted wave is

$$I_T = I_0 \cos^2 \theta.$$ \hspace{1cm} (2)

This relation is also known as Malus’ Law. If $\theta = \pi/2$ then the transmitted intensity is zero.

The polarization of the transmitted wave takes the orientation of the transmission axis of the polarizer. An interesting demonstration of this involves three polarizers. First we put two polarizers with their axes crossed. When you try to look through them you do not see anything. The first polarizer polarizes the light in an orientation that is orthogonal to the transmission axis of the second polarizer. However, when you put a third polarizer oriented at an intermediate angle in between the crossed polarizers you can see through! This is because the intermediate polarizer projects the light along its axis, an angle that is no longer orthogonal to the axis of the second polarizer.

**Question 4** You will be given three polarizers. Verify qualitatively all of the statements given above with two and three polarizers.

3.2 Polarization states of the photon

The polarization of photons can be expressed in terms of the polarization states $|H\rangle$ and $|V\rangle$. These represent the states of the photon when the polarization is horizontal and vertical, respectively. Because they represent orthogonal directions they cannot be expressed in terms of each other. Thus we can define them as members of an orthonormal basis, which obey the following relations:

$$\langle H|H \rangle = \langle V|V \rangle = 1$$ \hspace{1cm} (3)
$$\langle H|V \rangle = \langle V|H \rangle = 0.$$ \hspace{1cm} (4)
A photon linearly polarized along a direction that forms an angle $\theta$ relative to the horizontal can be expressed by a state $|\phi\rangle$ that is a linear superposition of states in the (H,V) basis

$$|\phi\rangle = \cos \theta |H\rangle + \sin \theta |V\rangle.$$ 

(5)

The (H,V) basis is only one of an infinite number of possible representations. Another basis could be (H$'$,V$'$), that is rotated from the (H,V) basis by an angle $\theta$, as shown in Fig. 3. The basis vectors $|H\rangle$ and $|V\rangle$ can be expressed in terms of the $|H'$\rangle and $|V'$\rangle basis vectors as

$$|H\rangle = \cos \theta |H'\rangle - \sin \theta |V'\rangle$$

(6)

$$|V\rangle = \sin \theta |H'\rangle + \cos \theta |V'\rangle.$$ 

(7)

Figure 3: Two bases to represent states of polarization.

**Question 5** Express $|H'\rangle$ and $|V'\rangle$ in terms of $|H\rangle$ and $|V\rangle$.

### 4 State projection with polarizers

A polarizer transmits photons polarized along its transmission axis and absorbs photons polarized perpendicular to its transmission axis. The polarization state of the photon after the polarizer is aligned with the axis of the polarizer. Thus the polarizer projects the state of the photon onto a state aligned with its transmission axis. If $|H'\rangle$ is aligned with the transmission axis of the polarizer then the action of the polarizer can be represented by the projection operator

$$P_{H'} = |H'\rangle\langle H'|.$$ 

(8)
The transmission of the photon initially in state $|\phi\rangle$ through the polarizer results in the projection

$$P_{H'}|\phi\rangle.$$ (9)

After the polarizer the photon is in state $|H'\rangle$. The probability amplitude for transmission and projection into state $|H'\rangle$ is

$$\langle H'|P_{H'}|\phi\rangle.$$ (10)

The probability that the photon gets transmitted is

$$P = |\langle H'|P_{H'}|\phi\rangle|^2 = |\langle H'|\phi\rangle|^2.$$ (11)

**Question 6** Suppose that the initial polarization state of the photon is $|H\rangle$. It is incident onto a polarizer with transmission axis oriented an angle $\theta$ relative to the horizontal.

1. What is the probability amplitude for transmission through the polarizer?
2. What is the probability that the photon is transmitted?
3. What is the state of the photon after transmission? Express your result in terms of the (H,V) basis.

### 4.1 Experiment 1: Transmission through one polarizer

We have the same setup as in last lab except that we will add a few polarizers.

**Procedure:**

1. The down converted photons are in state $|V\rangle$.
2. Put polarizer $A$ in the path of the signal beam before it reaches the detector, as shown in Fig. 1.
3. Use the data acquisition program “Geometric Phase.” This program records data in a start/stop mode. It stops after recording the detector counts for a specified amount of time. The stoppage allows time for the “operator” to adjust the polarizer to a new setting. When you are ready to proceed you hit space-bar in the computer keyboard. The computer then records the next data counts, repeating the cycle.
4. Record a scan as a function of the polarizer angle, starting from the zero reading of the polarizer (which should be the setting where the transmission axis is horizontal). Do this for two turns in increments of 15 degrees.
5. The measured coincidences are proportional to the probability of the photon being transmitted, $P = |\langle H_A|V\rangle|^2 = \sin^2 \theta_A$. Graph the data for the coincidences and find the settings of the polarizer that correspond to minima and maxima. Compare this with your expectations.
5 Sequential measurements

Suppose that we insert polarizer $B$ in front of polarizer $A$. The transmission axes of the polarizers $B$ and $A$ form angles $\theta_B$ and $\theta_A$ with the horizontal axis, respectively. The state of the photon past each polarizer is $|H_B\rangle$ and $|H_A\rangle$, respectively. The probability amplitude of the photon ending in state $|H_A\rangle$ is the amplitude of the initial state vector projected twice

$$P_A P_B |\phi\rangle = |H_A\rangle \langle H_A | H_B \rangle \langle H_B | \phi\rangle$$

or $\langle H_A | P_A P_B | \phi\rangle$. The probability will be given by

$$P = |P_A P_B |\phi\rangle|^2 = \langle \phi | P_B P_A P_A P_B | \phi\rangle.$$  \hfill (13)

5.1 Experiment 2: Transmission through two polarizers

1. Put polarizer $B$ in front of polarizer $A$.

2. Since polarizer $B$ projects the initial state $|V\rangle$ onto state $|H_B\rangle$, then when polarizer $A$ is rotated the maxima and minima of transmission will be in a different location—i.e., the quantum version of the three stacked polarizers. Thus the probability will be

$$P = ||H_A\rangle \langle H_A | H_B \rangle \langle H_B | V\rangle|^2 = \cos^2(\theta_A - \theta_B) \sin^2 \theta_B.$$  \hfill (14)

3. Set $\theta_B = \pi/4$ and take a scan of polarizer $A$, as you did in the first experiment.

4. Compare the graph with the expectation.