

The Quantum Eraser

Physics P371 – Quantum Mechanics

Spring 2007

In this lab we are going to use a very clever experiment as an exercise in state-vector algebra and to ponder further about the meaning of superposition and measurement.

1 Photonic polarization

1.1 Polarizers

A polarizer is easily understood classically. It provides the following function: when a linearly polarized electromagnetic wave is incident on a polarizer, the transmitted wave has an amplitude equal to the component of the field along the direction of the transmission axis of the polarizer and a polarization direction along that axis. If the Electric field vector of a wave of amplitude E_0 oscillates along a plane that forms an angle θ with the transmission axis of the polarizer, then the amplitude of the transmitted wave is

$$E_T = E_0 \cos \theta. \quad (1)$$

Since the intensity of the wave is proportional to the square of the amplitude, then the intensity of the transmitted wave is

$$I_T = I_0 \cos^2 \theta. \quad (2)$$

This relation is also known as Malus' Law. If $\theta = \pi/2$ then the transmitted intensity is zero.

The polarization of the transmitted wave takes the orientation of the transmission axis of the polarizer. An interesting demonstration of this involves three polarizers. First we put two polarizers with their axes crossed. When you try to look through them you do not see anything. The first polarizer polarizes the light in an orientation that is orthogonal to the transmission axis of the second polarizer. However, when you put a third polarizer oriented at an intermediate angle in between the crossed polarizers you can see through! This is because the intermediate polarizer projects the light along its axis, an angle that is no longer orthogonal to the axis of the second polarizer.

Question 1 You will be given three polarizers. Verify qualitatively all of the statements given above with two and three polarizers.

1.2 Polarization states of the photon

The polarization of photons can be expressed in terms of the polarization states $|H\rangle$ and $|V\rangle$. These represent the states of the photon when the polarization is horizontal and vertical, respectively. We also define the complex conjugates of these state vectors as

$$\langle H| = (|H\rangle)^* \quad (3)$$

and

$$\langle V| = (|V\rangle)^*. \quad (4)$$

Because $|H\rangle$ and $|V\rangle$ represent orthogonal directions they cannot be expressed in terms of each other. Thus we can define them as members of an orthonormal basis, which obey the following relations:

$$\langle H|H\rangle = \langle V|V\rangle = 1 \quad (5)$$

$$\langle H|V\rangle = \langle V|H\rangle = 0. \quad (6)$$

A photon linearly polarized along a direction that forms an angle θ relative to the horizontal can be expressed by a state $|\phi\rangle$ that is a linear superposition of states in the (H,V) basis

$$|\phi\rangle = a|H\rangle + b|V\rangle. \quad (7)$$

where $a = \cos\theta$ and $b = \sin\theta$. The probability amplitude that we measure the photon in state $|H\rangle$ is $\langle H|\phi\rangle = a$. the probability that we measure the polarization of the photon and find it in state $|H\rangle$ is

$$\mathbf{P} = |\langle H|\phi\rangle|^2 = \langle\phi|H\rangle\langle H|\phi\rangle = a^*a. \quad (8)$$

The (H,V) basis is only one of an infinite number of possible representations. Another basis could be (H',V') , that is rotated from the (H,V) basis by an angle θ , as shown in Fig. 1. The basis vectors $|H\rangle$ and $|V\rangle$ can be expressed in terms of the $|H'\rangle$ and $|V'\rangle$ basis vectors as

$$|H\rangle = \cos\theta|H'\rangle - \sin\theta|V'\rangle \quad (9)$$

$$|V\rangle = \sin\theta|H'\rangle + \cos\theta|V'\rangle. \quad (10)$$

Question 2 Express $|H'\rangle$ and $|V'\rangle$ in terms of $|H\rangle$ and $|V\rangle$.

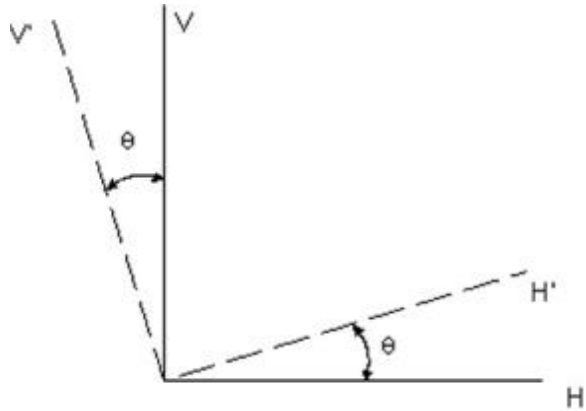


Figure 1: Two bases to represent states of polarization.

2 State projection with polarizers

A polarizer transmits photons polarized along its transmission axis and absorbs photons polarized perpendicular to its transmission axis. The polarization state of the photon after the polarizer is aligned with the axis of the polarizer. Thus the polarizer *projects* the state of the photon onto a state aligned with its transmission axis. If $|H'\rangle$ is aligned with the transmission axis of the polarizer then the action of the polarizer can be represented by the projection operator

$$P_{H'} = |H'\rangle\langle H'|. \quad (11)$$

The transmission of the photon initially in state $|\phi\rangle$ through the polarizer results in the projection

$$P_{H'}|\phi\rangle. \quad (12)$$

After the polarizer the photon is in state $|H'\rangle$. The probability amplitude for transmission and projection into state $|H'\rangle$ is

$$\langle H'|P_{H'}|\phi\rangle. \quad (13)$$

The probability that the photon gets transmitted is

$$\mathbf{P} = |P_{H'}|\phi\rangle|^2 \quad (14)$$

Note that $P_{H'}P_{H'} = P_{H'}$ due to the normalization conditions of Eqs. 10, so the probability can be written as

$$\mathbf{P} = \langle\phi|P_{H'}|\phi\rangle. \quad (15)$$

Question 3 Suppose that the initial polarization state of the photon is $|H\rangle$. It is incident onto a polarizer with transmission axis oriented an angle θ relative to the horizontal.

1. What is the probability amplitude for transmission through the polarizer?
2. What is the probability that the photon is transmitted?
3. What is the state of the photon after transmission? Express your result in terms of the (H,V) basis.

2.1 Experiment 1: Transmission through one polarizer

We have the same setup as in last lab except that we will add a few polarizers. To get better data we will divert the signal photons to a detector located before the interferometer. You must first maximize the signal on each detector by tilting the relevant experimental parameters.

Procedure:

1. The orientation of the fibers that take the light to the detectors.
2. The orientation of the beam splitter and mirror that deflect the light to the idler and signal fibers, respectively.
3. The tilt of the down-converter crystal.
4. Put polarizer *A* in the path of the signal beam before it reaches the mirror that steers the signal photons into the fiber, as shown in Fig. 2.

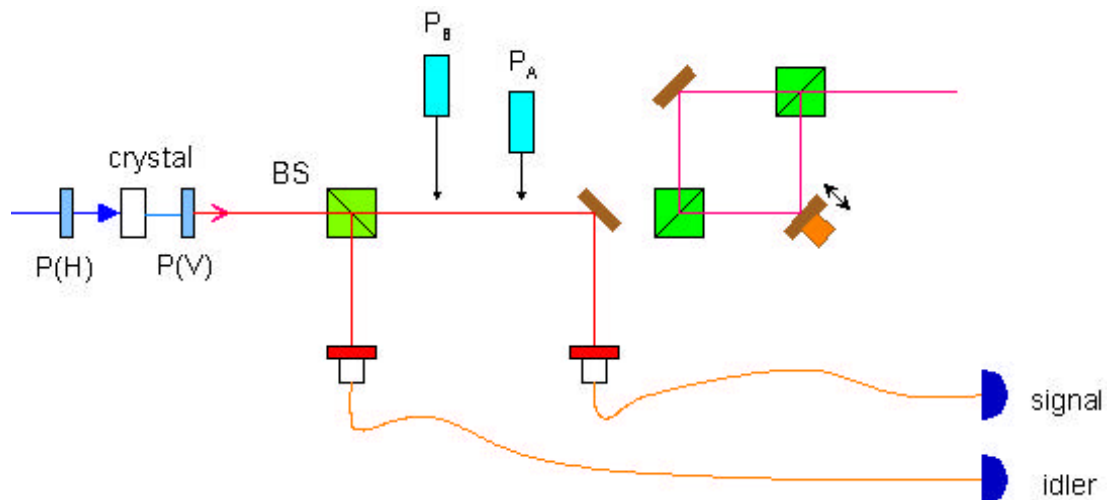


Figure 2: A schematic of the apparatus for Experiment 1.

5. Use the data acquisition program “Geometric Phase Counter Ray2.” This program records data in a start/stop mode. It stops after recording the detector counts for a specified amount of time. The stoppage allows time for the “operator” to adjust the polarizer to a new setting. When you are ready to proceed you hit space-bar in the computer keyboard. The computer then records the next data counts, repeating the cycle.

Record a scan as a function of the polarizer angle. Do this for two turns in increments of 15 degrees.

6. The measured coincidences are proportional to the probability of the photon being transmitted,

$$\mathbf{P} = \langle V | H_A \rangle \langle H_A | V \rangle = \sin^2 \theta_A.$$

Graph the data for the coincidences and find the settings of the polarizer that correspond to minima and maxima. Compare this with your expectations.

3 Sequential measurements

Suppose that we insert polarizer B in front of polarizer A . The transmission axes of the polarizers B and A form angles θ_B and θ_A with the horizontal axis, respectively. The state of the photon past each polarizer is $|H_B\rangle$ and $|H_A\rangle$, respectively. The probability amplitude of the photon ending in state $|H_A\rangle$ is the amplitude of the initial state vector projected twice

$$P_A P_B |\phi\rangle = |H_A\rangle \langle H_A | H_B \rangle \langle H_B | \phi \rangle. \quad (16)$$

Question 4 Show that the probability will be given by

$$\mathbf{P} = \langle \phi | P_B P_A P_B | \phi \rangle. \quad (17)$$

3.1 Experiment 2: Transmission through two polarizers

1. Put polarizer B in front of polarizer A . This projects state $|V\rangle$ onto state $|H_B\rangle$
2. Since polarizer B projects the initial state $|V\rangle$ onto state $|H_B\rangle$, then when polarizer A is rotated the maxima and minima of transmission will be in a different location—i.e., the quantum version of the three stacked polarizers. Thus from Eq. 17 the probability will be

$$\mathbf{P} = \langle V | H_B \rangle \langle H_B | H_A \rangle \langle H_A | H_A \rangle \langle H_A | H_B \rangle \langle H_B | V \rangle \quad (18)$$

$$= \cos^2(\theta_A - \theta_B) \sin^2 \theta_B. \quad (19)$$

3. Set $\theta_B = \pi/4$ and take a scan of polarizer A , as you did in the first experiment.
4. Compare the graph with the expectation.

4 The Quantum Eraser

This experiment entails three parts. We use polarizers to project the state of the photons in such a way that we can define whether we should get interference or not. As we will see later, this experiment also has deeper implications on the significance of quantum interference.

4.1 Experiment 3: Quantum interference

1. Set up the interferometer so that there are two waveplates, one on each arm of the interferometer, as shown in Fig. 3.

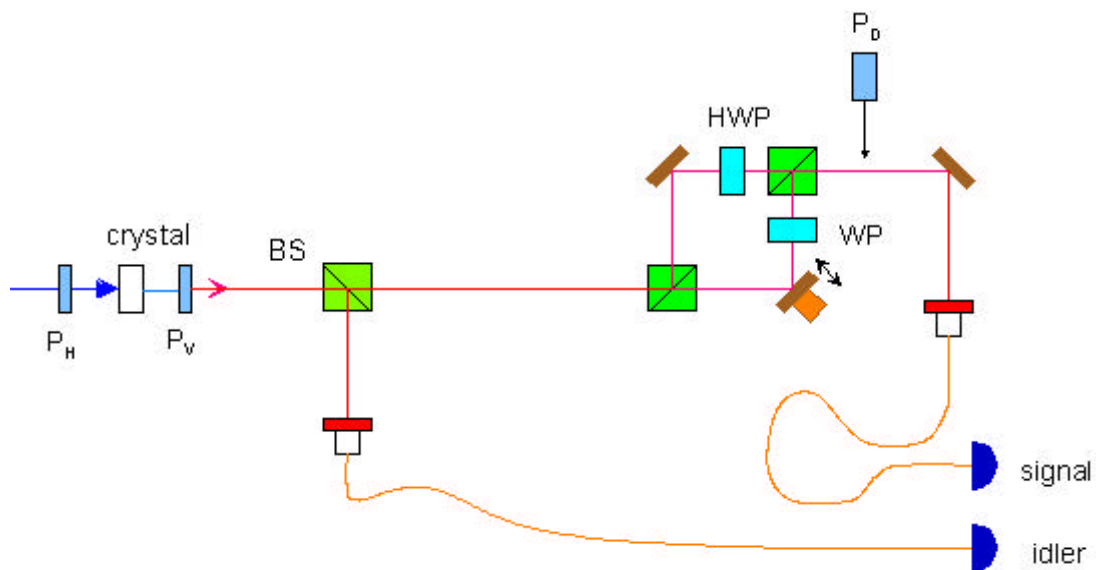


Figure 3: A schematic of the apparatus.

2. Set both polarizers so that the transmitted state is $|V\rangle$. That is, set $\theta_A = \theta_B = 90^\circ$. The probability of detecting a photon past the interferometer is

$$P = |rt(|V\rangle + e^{i\delta}|V\rangle)|^2 = 2r^*rt^*t(1 + \cos \delta) = \frac{1}{2}(1 + \cos \delta), \quad (20)$$

where, as before, δ is the phase due to the difference in length between the two arms, and $r = i/\sqrt{2}$ and $t = 1/\sqrt{2}$ are the reflection and transmission coefficients of the beam splitters, respectively.

3. Take a piezo scan to obtain the interference of the photons going through the interferometer. Use the Dynamical Phase program.
4. Up to now the polarization state of the photon taking either path is the same, i.e., $|V\rangle$. the paths are thus indistinguishable. Now rotate the half-waveplate to 45° leaving the other one unchanged. Because the new state after the half-waveplate is $|H\rangle$, it is orthogonal to the state of the light coming through the other arm. The paths are distinguishable and therefore we should not get any interference. Find the probability, similar to Eq. 20, to show that there is no interference.

Notice that we don't even try to measure the polarization of the paths. Interference disappears even if *in principle* we can obtain the path information.

5. Take a piezo scan to verify this.
6. Now place a polarizer after the interferometer with its transmission axis set to 45° (i.e., state $|D\rangle$).

Note that the diagonal state $|D\rangle$ can be expressed as

$$|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad (21)$$

and the antidiagonal state can be represented by

$$|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle) \quad (22)$$

The polarizer projects the state of the light onto state $|D\rangle$ What do you expect in terms of interference? The probability is now

$$P = |rt(P_D|H\rangle + e^{i\delta} P_D|V\rangle)|^2. \quad (23)$$

Simplify this equation.

7. Take a scan and compare the results with your predictions.

The last part is called the “quantum eraser.” By placing the polarizer after the interferometer we erase the path-labeling information, and thus we regain interference. It is striking that we decide whether to get or not the interference *after* the photon goes through the interferometer. Such a scheme has been argued extensively in the literature, and has been labeled a “delayed choice” experiment.

Finally, note that we erase the path-distinguishing information by manipulating the states of the system. This is in contrast to Feynman's double-slit experiment where the interference was destroyed by disturbing the system in an uncontrollable and unpredictable way.