Identical Photons

Physics P371 – Quantum Mechanics

Spring 2009

In this lab we are going to study a new kind of interference. We still have two identical photons produced by parametric down conversion, but in contrast with the previous experiments both photons go through the interferometer. In this case we need to specify the state of two particles. Since they are bosons this wavefunction will have to be symmetric. The interference pattern will reflect the symmetry of the wavefunction of the photon pair.

1 Experimental considerations

1.1 Apparatus

Figure 1 shows a schematic of the apparatus that we will use. It is considerably simpler from the hardware standpoint. However, it is rich in quantum physics. First we have that photon pairs are produced by parametric down-conversion. The down-converted photons leave the crystal in the same direction. Both photons then enter the interferometer. We will detect photons in two modes: (1) when they come out of the different output ports of the interferometer, and (2) when both photons exit the same output port of the interferometer.

Figure 1: Apparatus for the interference of identical photons.
When the photons leave together they encounter a beam splitter. The detectors are placed at either output of the beam splitter. We will record photons detected by each detector plus the coincident events. Since the photons will be indistinguishable from each other, it no longer make sense of calling them the idler and the signal, so we will call them 1 and 2.

Before we set the stage for two-photon interference, let’s recall the situation for single-photon interference.

## 2 Four Ways to go through the Interferometer

The first step in figuring out the interference is the possible ways in which the two photons can go through the interferometer. These are listed in Fig. 2:

- **Case A**: both photons go through arm 1,
- **Case B**: both photons go through arm 2,
- **Case C**: the signal goes through arm 1 and the idler through arm 2,
- **Case D**: the signal goes through arm 2 and the idler through arm 1.

![Figure 2: Ways in which two photons go through the interferometer.](image)

### 2.1 Probability Amplitudes

Suppose that the difference in length of the two arms is small enough that when the photon goes through the interferometer it is not possible to distinguish which path it takes. If the length of the arms are \( \ell_1 \) and \( \ell_2 \), and if the two photons have a wave...
number \( k_0 = 2\pi/\lambda_0 \), then the phases that the photon acquires in going through the arms 1 and 2 of the interferometer are \( \delta_1 = k_0\ell_1 \) and \( \delta_2 = k_0\ell_2 \), respectively.

**Question 1** Write down the probability amplitudes for detecting the two photons past the interferometer for each of the four cases above. Use \( r = i/\sqrt{2} \) and \( t = 1/\sqrt{2} \) for the reflection and transmission probability amplitudes, respectively, at each beam splitter.

**Question 2** Show that the probability for the two photons leaving the interferometer is

\[
P = \frac{1}{4}(1 + \cos \delta)^2,
\]

where for convenience we have defined \( \delta = \delta_1 - \delta_2 \).

Notice that the probability of Eq. 1 oscillates between 0 and 1 as \( \delta \) is varied. In a past lab we obtained the probability for a single photon going through the interferometer to be:

\[
P = \frac{1}{2}(1 + \cos \delta).
\]

This one also oscillates between 0 and 1, but in a different way.

**Question 3** Use your favorite graphing package to make graphs of Eqs. 1 and 2. Comment on their difference.

### 2.2 State-Vector Approach

The formal way to treat this problem is using state-vector mechanics. If an incoming photon is in state \( |i\rangle \), upon reaching the first beam splitter the wave function splits into two parts, on for each arm of the interferometer:

\[
|\psi_1\rangle = r|\phi_y\rangle + t|\phi_x\rangle,
\]

where we have called \( |\phi_x\rangle \) and \( |\phi_y\rangle \) the spatial wavefunction for a photon traveling along the \( x \) and \( y \) directions, respectively. The mirror then flips those subsequently. In traveling through the length of the arms, the state of the light going through the arms gain a phase,

\[
|\phi_x\rangle \rightarrow e^{i\delta_1}|\phi_x\rangle
\]

\[
|\phi_y\rangle \rightarrow e^{i\delta_2}|\phi_y\rangle.
\]

Upon reaching the beam splitter acts upon the light again.
Question 4 Compute the final state of the light.

Question 5 Calculate the probability that the light ends in the final state $|\phi_x\rangle$.

When the two identical photons reach the interferometer their wavefunction can be described by a symmetric product wavefunction

$$|\Psi\rangle = |\phi_x\rangle_1|\phi_x\rangle_2,$$

(6)

The initial wavefunction thus transforms to

$$|\psi\rangle = \alpha|\phi_x\rangle_1|\phi_x\rangle_2 + \beta|\phi_x\rangle_1|\phi_y\rangle_2 + \gamma|\phi_y\rangle_1|\phi_x\rangle_2 + \epsilon|\phi_y\rangle_1|\phi_y\rangle_2,$$

(7)

where $\alpha$, $\beta$, $\gamma$ and $\epsilon$ are complex coefficients.

Question 6 Show that the probability amplitude $\alpha$ is $r^2t^2(1 + e^{i\delta})^2$.

Question 7 Find $\beta$.

Question 8 Find $\gamma$, which turns out to be equal to $\epsilon$.

Notice then that when the light leaves separate ports of the interferometer it is in a symmetric state

$$\Psi\rangle = \frac{1}{\sqrt{2}}(|\phi_x\rangle_1|\phi_y\rangle_2 + |\phi_y\rangle_1|\phi_x\rangle_2)$$

(8)

This is a very important result in the quantum mechanics of identical particles. Those with integer spins (bosons) are described by symmetric wavefunctions, and those with fractional spin (fermions) are described by antisymmetric wavefunctions.

2.3 Procedure Part I

1. Take a piezo scan from 0 to 3 V with 0.02 V increments.

2. Make a graph of the singles for each detector plus the coincidences.

3. Based on Eqs. 1 and 2 explain the differences and similarities between the three graphs.
3 Biphoton Interference

Now we need to stop and wait until the other group is done with this part. Connect the fiber after the other \((y)\) output port of the interferometer to one of the detector inputs.

**Question 9** Show that the probability of detecting coincidences is given by

\[ P = \frac{1}{4}(1 + \cos 2\delta). \tag{9} \]

3.1 Procedure Part II

1. Take a scan like before.

2. Graph the three sets of data (singles and coincidences) and compare them with the expectation. A “curve ball” might be thrown at you by the two beam splitters not being identical (e.g., \(r_1 = r_2 f\), where \(f\) is a complex number less than one).

4 Distinguishable Paths

Now return to the previous detector connection. We now increase the length of one of the arms until we make the paths distinguishable. Analyze Fig. 2. In the experiments we have a beam splitter after the interferometer. Half of the time the pair of photons split at the last beam splitter to reach the detectors. This can be seen by applying Eq. 3 in Eq. 6.

**Question 10** Do all cases of Fig. 2 become distinguishable? Explain.

The probability of detecting coincidences is given by

\[ P = \frac{1}{8}(1 + \cos 2\delta) + \frac{1}{8}. \tag{10} \]

**Question 11** If the uncertainty in the wavelength \(\lambda_0\) of the down-converted photons is \(\Delta\lambda\), the coherence length is \(\ell_c = \lambda_0^2/\Delta\lambda\). Calculate \(\ell_c\). What is the meaning of \(\ell_c\)?

**Question 12** If the difference in length of the two arms is \(\ell\), what condition must \(\ell\) satisfy for the two arms to be quantum mechanically indistinguishable?
Question 13 If the wavelength of the laser is 804 nm and the bandwidth of the downconverted light is 40 nm, what is the coherence length of the down-converted light?

Question 14 What is the coherence of the pump laser if it has a bandwidth of 10 nm?

4.1 Procedure Part III

1. Changing the setting of the micrometer by the smallest subdivision changes the length of arm 1 by 35 µm. Record the current setting of the micrometer. Using the micrometer on the translation stage decrease the length of Arm 1 by 35 µm in 3 intervals and record a scan for each time. Double the number of seconds per point. Observe the differences from the previous scan.

2. Decrease the length of the interferometer again by 105 µm (3 subdivisions) and record a scan. Increase the number of seconds per scan. This new setting may require a realignment of the interferometer by the instructor.

3. Graph the first and last scan. Does the data shift in shape from Eq. 1 to Eq. 10?

4. The last scan shows interference. However, the two arms are distinguishable (i.e., single photon interference goes away), so this must be a new kind of interference. What aspect of the detection of the two photons is indistinguishable?

The behavior of the photon pair as a whole is quite striking. Both photons behave as one photon with half the wavelength, and thus the name biphoton.