Photon Wavepackets

Physics P371 – Quantum Mechanics

Spring 2005

In this lab we are going to investigate more aspects about the properties of correlated photons. In a first part we will understand the timing aspects of the experiments. Then we will see the interference of photons in a more realistic sense, by realizing that the photons in this experiments are not infinite wave-trains but wavepackets. The finite length of wavepackets puts restrictions on the interference of photons. We will use this one step further to show how the photons produced by parametric down-conversion are correlated in energy.

1 Timing of the Experiments

When down-conversion occurs the energy of the pump photon $E_p$ is converted into that of two photons, the signal and the idler, $E_s$ and $E_i$. Energy conservation requires

$$E_p = E_s + E_i.$$ (1)

This relation forces us to conclude that the down-converted photons are produced simultaneously. We use this to our advantage to study the interference of single photons, and as we will see here, to have some degree of “remote” control over the interference pattern.

To detect photon pairs we must detect photons that were created at the same time. If they travel the same distance from the crystal to the detectors, then the electronic pulses signaling the detection of photons will be produced simultaneously. If one photon arrives earlier than the other one then the electronic pulses of photon pairs will be delayed from each other. Thus, sometimes is more practical to record electronic pulse pairs with a specific delay. It relieves the requirement of adjusting the apparatus so that the photons arrive to the detectors simultaneously.

We use an electronic module, called time-to-amplitude converter (TAC), to record the signal from photon pairs that are delayed from each other. We can further specify a range of delays that we will take as the signal of a pair. The TAC is very easy to understand. It takes the pulse of the photon that arrives earlier and uses it to trigger a timer. The partner photon that arrives later is used to stop the timer. The timer
count is converted to a voltage proportional to the timer count. The pulse then can proceed to a second section of the apparatus for pulse-height analysis. We will use a multichannel analyzer (MCA) to view a histogram of pulses as a function of their height (i.e., delay). This MCA is in a second PC. It is fed by the TAC output of the module. With the use of a single-channel analyzer (SCA) we can select what range of pulse heights we want to accept as “coincidences.” The SCA is also contained in the module. The SCA output of the module gives out a pulse whenever the delay between the start and stop pulses is within the times $T$ and $T + \Delta T$. The latter values, $T$ and $T + \Delta T$ are specified by controls on the front panel.

![Diagram of electronics](image.png)

Figure 1: Partial schematic diagram of the electronics.

1.1 Procedure

1. With the room lights turned-on make a complete diagram of the wiring of the experiment.

2. Measure the difference in length of the cables for the signal and idler.

3. Measure the extra distance that the signal photons have travel to the detector.

4. The TAC has three control knobs/dials: Range, $T$ and $\Delta T$. The range specifies the maximum range of time that the timer of the TAC will use for start/stop pulse pairs. $T$ and $\Delta T$ are specified as a fraction of the Range. Record the settings of these controls.
5. Now set $T = 0$ and $\Delta T = 10$. Block one of the arms of the interferometer and record all three counter readings.

6. Acquire a histogram and save it to a file.

7. The scale of the histogram may not be linear. Use the digital-delay-generator to acquire a histogram with known delays. Record the delays that you use, and then save the file.

8. In your report graph the first histogram and use interpolation to find the delay between the idler and the signal electronic pulses that reach the TAC module.

9. Use this information to find the speed of the pulses inside the cable as a fraction of the speed of light.

10. Reconnect the cables and return the settings to their original values.

11. Record the coincidences with the reduced time window.

2 Photon Wavepackets

The photons produced by parametric down conversion obey Eq. 1. However, this equation does not state the energy of each individual photon. Thus, the down-converted photons may have a spread in energy, as long as their sum obeys Eq. 1. If we state this in terms of the photon wavenumber $k = E/(hc) = 2\pi/\lambda$ then we can talk about the spread in energy in terms of $k$.

If we consider the signal photon, its wave-function must account for the spread of the wavenumbers. In the notation done in class we have that it is given by

$$\Psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \phi(k)e^{ikx}dk,$$  \hspace{1cm} (2)

where $\phi(k)$ is the normalized amplitude of the component with wavenumber $k$. It satisfies the normalization condition

$$\int_0^\infty \phi(k)^*\phi(k)dk = 1$$ \hspace{1cm} (3)

Thus, the photon can be thought of as a wavepacket.

**Question 1** Using Eq. 3 show that the stationary wave function of Eq. 2 is normalized.

In our experiment $\phi(k)$ is defined by the filter in front of the detector, which allows light with wavenumbers between $k_0 - \Delta k/2$ and $k_0 + \Delta k/2$, with $k_0 = k_p/2 = E_p/(2hc)$. A simplified form of $\phi(k)$ is shown in Fig. 2.
Thus, combining Eq. 2 with the form of \( \phi(k) \) given in Fig. 2 we get a wavefunction given by

\[
\Psi(x, 0) = \frac{1}{\sqrt{2\pi \Delta k}} \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} e^{ikx} dk.
\] (4)

The part of the wavefunction where the photon is going through the straight output of the interferometer is

\[
\Psi(x, 0) = \frac{1}{\sqrt{2\pi \Delta k}} \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} rt(e^{-i\ell_1} + e^{-i\ell_2}) e^{ikx} dk,
\] (5)

where now the effect of the uneven length of the arms \( \ell_1 \) and \( \ell_2 \) is to modify the phases for the two paths. The reflection coefficients are given by \( r = i/\sqrt{2} \) and \( t = 1/\sqrt{2} \).

It can be shown that the probability of detecting the photon after the Mach-Zehnder interferometer (see Fig. 3) is

\[
P = \int_{-\infty}^{+\infty} \Psi^*(x, 0)\Psi(x, 0)dx = \frac{1}{4\Delta k} \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} (e^{i\ell_1} + e^{i\ell_2})(e^{-i\ell_1} + e^{-i\ell_2})dk.
\] (6)

**Question 2** Show that Eq. 6 reduces to

\[
P = \frac{1}{2} [1 + \text{sinc}\frac{\Delta k\Delta \ell}{2} \cos \delta],
\] (7)

where \( \text{sinc}(x) = \frac{\sin x}{x} \), and \( \delta = k_0\Delta \ell \).

The \text{sinc}(x) function has the property that when \( x \to 0 \) then \( \text{sinc}(x) = 1 \), and when \( x \gg 1 \) then \( \text{sinc}(x) \to 0 \). The \text{sinc}(x) function is a result of the square shape of the amplitude \( \phi(k) \). In general it would be a function \( V \) that has the limits of 1 and 0 when \( x = 0 \) and \( x \gg 1 \), respectively. This function is called the *visibility*, which is the amplitude of the oscillations in the probability.
Figure 3: A schematic of the apparatus.

$V$ can be determined experimentally using

$$V = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}},$$

(8)

where $N_{\text{max}}$ and $N_{\text{min}}$ are the maximum and minimum counts in the oscillations of the recorded oscillations ($\propto P$).

When $V = 1$ we say that we have full fringe visibility, and the oscillations in the probability [i.e., $P = (1 + \cos \delta)/2$] represent the indistinguishability of the interferometer paths. When $V(x) = 0$ there are no fringes (i.e., $P = 1/2$).

If $\Delta \ell = 0$ (i.e. the arms have equal length), then we have $V = 1$ and full visibility. Conversely, when $\Delta \ell \gg 2/\Delta k$ then $V \rightarrow 0$. What does the latter mean? In class we saw that for a wave packet the uncertainty principle says that

$$\hbar \Delta k \Delta x \geq \hbar,$$

(9)

where $\Delta x$ is the uncertainty in the position of the wavepacket (see Fig. 4). Thus, we can see $\Delta x$ as the width of the wavepacket. For simplicity we will rename $\ell_c \equiv \Delta x$ the “coherence length,” a term used more frequently in optics.

Figure 4: A representation of a wavepacket.

The filters are rated by the $wavelength$ interval $\Delta \lambda$ that they allow.
Question 3  Express $\ell_c$ in terms of $\Delta \lambda$ and $\lambda$.

Question 4  If the filter in front of the detector has a width of $\Delta \lambda = 10$ nm, what is the value of $\ell_c$?

Question 5  What is the temporal width of the wave-packet?

Question 6  The detection efficiencies make the recorded coincidences to be about 0.1 of the pairs that are going through the interferometer. Given the coincidence count rate, estimate the average number of photons with a partner going through the interferometer at any given time. The interferometer has a total length of about 150 mm.

Question 7  The micrometer in Fig. 3 pushes the mirror in a diagonal direction ($45^\circ$ to the arm axes). By how much does the length of the arm change when the micrometer moves by 10 $\mu$m?

2.1 Procedure

1. Take a scan of the coincidences as a function of the arm length difference using the piezo driven mirror, similar to the scans that we took last week.

2. Increase the micrometer reading by 10 $\mu$m (smallest interval) and retake the scan.

3. Repeat the previous step until the fringes are gone.

4. Make a graph of $V$ vs. $\Delta \ell$.

5. Explain the lack of fringes in terms of the distinguishability of the paths.
   What would happen if we put a 1-nm filter in front of the detector? The photon transmitted through this filter will have a different wave function because its distribution in $k$ is now narrower by a factor of ten. If $\Delta \lambda$ decreases by a factor of 10 then $\Delta k$ also decreases by the same factor ($\Delta k = 2\pi \Delta \lambda / \lambda^2$). However, $\ell_c$ must increase by a factor of ten, via the uncertainty principle.
6. How does the last value of \(\Delta \ell\) compare with the new value of \(\ell_c\)? Indeed we may recover the fringes!

Now here is something stunning. The two photons (signal and idler) are correlated in energy by Eq. 1. If one photon has a wavenumber \(k_0 - k\) then the other one has a corresponding wavenumber \(k_0 + k\). We have not covered correlated states of two functions yet, but strictly, our wavefunction should be the product of the wave functions of the two photons

\[
\Psi(x_i, x_s) = \frac{1}{2\pi \sqrt{2}} \int \phi_i(k_0 - k)e^{i(k_0 - k)x_i}\phi_s(k_0 + k)e^{i(k_0 + k)s} dk. \quad (10)
\]

Thus the wave-packet of the photon going through the interferometer depends on the distribution of \(k\)'s for both photons [i.e., on the product \(\phi_i(k_0 - k)\phi_s(k_0 + k)\)]. Therefore, it does not matter on which detector we put the 1-nm filter!

7. Put the 1-nm filter in front of the idler detector. Retake a scan.

8. Explain what you observe and write your conclusions.